Higher derivative corrections to Wess-Zumino action of Brane-Antibrane systems

Mohammad R. Garousi

Department of Physics, Ferdowsi university, P.O. Box 1436, Mashhad, Iran
Institute for Studies in Theoretical Physics and Mathematics (IPM)
P.O.Box 19395-5531, Tehran, Iran

Abstract

By explicit calculation, we show that the expansion of the disk level S-matrix element of one RR field, two open string tachyons and one gauge field that has been recently found corresponds to the derivative expansion of the Wess-Zumino action of D-brane-anti-D-brane systems.

1 Introduction

Study of unstable objects in string theory might shed new light in understanding properties of string theory in time-dependent backgrounds [1, 2, 3, 4, 5, 6, 7]. Generally speaking, source of instability in these processes is appearance of some tachyonic modes in the spectrum of these objects. It then makes sense to study them in a field theory which includes those modes. In this regard, it has been shown by A. Sen that an effective action of the Dirac- Born-Infeld type proposed in [8, 9, 10, 11] can capture many properties of the decay of non-BPS D_p-branes in string theory [2, 3]. This action has been found in [9] by studying the S-matrix element of one graviton and two tachyons.

Recently, unstable objects have been used to study spontaneous chiral symmetry breaking in holographic model of QCD [12, 13, 14]. In these studies, flavor branes introduced by placing a set of parallel branes and antibranes on a background dual to a confining color theory [15]. Detailed study of brane-anti-brane system reveals when branes separation is smaller than the string length scale, the spectrum has two tachyonic modes [16]. The effective action should then include these modes as they are the most important ones which rule the dynamics of the system.

The effective action of a $D_p\bar{D}_p$-brane in Type IIA(B) theory should be given by some extension of the DBI action and the WZ terms which include the tachyon fields. The DBI part may be given by the projection of the effective action of two non-BPS D_p-branes in Type IIB(A) theory with $(-1)^{F_L}$ projection [17]. We are interested in this paper in the appearance of tachyon, gauge field and the RR field in these actions. These fields appear in the DBI part as the following [18]:

$$S_{DBI} \;=\; -T_p \int d^{p+1}\sigma \mathrm{Tr}\left(V(\mathcal{T})\sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab} + 2\pi\alpha' D_a\mathcal{T} D_b\mathcal{T})}\right) \;, \qquad (1)$$

where T_p is the p-brane tension. The trace in the above action should be completely symmetric between all matrices of the form $F_{ab}, D_a\mathcal{T}$, and individual $\mathcal{T}$ of the tachyon potential. These matrices are

$$F_{ab} = \begin{pmatrix} F^{(1)}_{ab} & 0 \\ 0 & F^{(2)}_{ab} \end{pmatrix},\; D_a\mathcal{T} = \begin{pmatrix} 0 & D_aT \\ (D_aT)^* & 0 \end{pmatrix},\; \mathcal{T} = \begin{pmatrix} 0 & T \\ T^* & 0 \end{pmatrix} \qquad (2)$$

where $F^{(i)}_{ab} = \partial_a A^{(i)}_b - \partial_b A^{(i)}_a$ and $D_aT = \partial_aT - i(A^{(1)}_a - A^{(2)}_a)T$. The tachyon potential which is consistent with S-matrix element calculations has the following expansion:

$$V(\mathcal{T}) \;=\; 1 + \pi\alpha' m^2\mathcal{T}^2 + \frac{1}{2}(\pi\alpha' m^2\mathcal{T}^2)^2 + \cdots$$

where m^2 is the mass squared of tachyon, *i.e.*, $m^2 = -1/(2\alpha')$. The above expansion is consistent with the potential $V(\mathcal{T}) = e^{\pi\alpha' m^2 \mathcal{T}^2}$ which is the tachyon potential of BSFT [19]. This action has the following expansion:

$$\mathcal{L}_{DBI} = -2T_p - T_p(2\pi\alpha')\left(m^2|T|^2 + DT\cdot(DT)^* - \frac{\pi\alpha'}{2}\left(F^{(1)}\cdot F^{(1)} + F^{(2)}\cdot F^{(2)}\right)\right) + \cdots \quad (3)$$

where dots refers to the terms which have more than two fields.

The WZ term describing the coupling of RR field to tachyon and gauge field of brane-anti-brane is given by [20, 21, 22]

$$S_{WZ} = \mu_p \int_{\Sigma_{(p+1)}} C \wedge \mathrm{STr}\, e^{i2\pi\alpha'\mathcal{F}} \quad (4)$$

where the curvature of the superconnection is defined as:

$$\mathcal{F} = d\mathcal{A} - i\mathcal{A}\wedge\mathcal{A} \quad (5)$$

the superconnection is

$$i\mathcal{A} = \begin{pmatrix} iA^{(1)} & \beta T^* \\ \beta T & iA^{(2)} \end{pmatrix},$$

where β is a normalization constant. If one uses the tachyon DBI action (1) for describing the dynamics of the tachyon field then the normalization of tachyon in the WZ action (4) has to be [23]

$$\beta = \frac{1}{\pi}\sqrt{\frac{2\ln(2)}{\alpha'}} \quad (6)$$

The "supertrace" in (4) is defined by

$$\mathrm{STr}\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \mathrm{Tr}\, A - \mathrm{Tr}\, D\ .$$

Using the multiplication rule of the supermatrices [21]

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}\cdot\begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} = \begin{pmatrix} AA' + (-)^{c'}BC' & AB' + (-)^{d'}BD' \\ DC' + (-)^{a'}CA' & DD' + (-)^{b'}CB' \end{pmatrix} \quad (7)$$

where x' is 0 if X is an even form or 1 if X is an odd form, one finds that the curvature is

$$i\mathcal{F} = \begin{pmatrix} iF^{(1)} - \beta^2|T|^2 & \beta(DT)^* \\ \beta DT & iF^{(2)} - \beta^2|T|^2 \end{pmatrix},$$

where $F^{(i)}=\frac{1}{2}F^{(i)}_{ab}dx^a\wedge dx^b$ and $DT=[\partial_a T-i(A^{(1)}_a-A^{(2)}_a)T]dx^a$. Using the expansion for the exponential term in the WZ action (4), one finds many different terms. The terms which involve at most three open string fields are the following:

$$
\begin{aligned}
\mu_p(2\pi\alpha')C\wedge \mathrm{STr}\, i\mathcal{F} &= \mu_p(2\pi\alpha')C_{p-1}\wedge(F^{(1)}-F^{(2)}) \qquad (8)\\
\frac{\mu_p}{2!}(2\pi\alpha')^2 C\wedge \mathrm{STr}\, i\mathcal{F}\wedge i\mathcal{F} &= \frac{\mu_p}{2!}(2\pi\alpha')^2 C_{p-3}\wedge\left\{F^{(1)}\wedge F^{(1)}-F^{(2)}\wedge F^{(2)}\right\}\\
&\quad +C_{p-1}\wedge\left\{-2\beta^2|T|^2(F^{(1)}-F^{(2)})+2i\beta^2 DT\wedge(DT)^*\right\}\\
\frac{\mu_p}{3!}(2\pi\alpha')^3 C\wedge \mathrm{STr}\, i\mathcal{F}\wedge i\mathcal{F}\wedge i\mathcal{F} &= \frac{\mu_p}{3!}(2\pi\alpha')^3 C_{p-3}\left\{3i\beta^2(F^{(1)}+F^{(2)})\wedge DT\wedge(DT)^*\right\}
\end{aligned}
$$

The coupling of one RR field C_{p-1}, two tachyons and one gauge field in the above terms can be combined into the following form:

$$
\begin{aligned}
&-\beta^2\mu_p(2\pi\alpha')^2\int_{\Sigma_{(p+1)}}C_{(p-1)}\wedge\left\{d(A^{(1)}-A^{(2)})TT^*-(A^{(1)}-A^{(2)})d(TT^*)\right\}\\
&\qquad = -\beta^2\mu_p(2\pi\alpha')^2\int_{\Sigma_{p+1}}H_{(p)}\wedge(A^{(1)}-A^{(2)})TT^* \qquad (9)
\end{aligned}
$$

This combination actually appears naturally in the S-matrix element in the string theory side [23]. It has been shown in [23] that the effective actions (1) and (4) are consistent with an expansion of the S-matrix element of one RR, two tachyons and one gauge field. In the present paper, we would like to show that the expansion found in [23] is in fact the momentum expansion, *i.e.,* the expansion which is consistent with the derivative expansion of the field theory of brane-anti-brane system. We will show this by explicitly calculating the higher derivative terms of the WZ field theory, *i.e.,* (29), (35), and (44).

An outline of the rest of paper is as follows. In the next section, we study the momentum expansion of the S-matrix element of one RR and two tachyons, and the S-matrix element of one RR and two gauge fields. We shall find in this section the higher derivative extension of the coupling in the second line of (8) and the coupling in the last term in the third line of (8). In section 3, we study the momentum expansion of the S-matrix element of one RR, two tachyons and one gauge field. We shall find the higher derivatives extension of the coupling in the last line of (8) and the coupling in the first term in the third line of (8). In this section we will also find a class of higher derivative terms, *i.e.,* (44) which has at least four derivatives, hence, they are not higher derivative extension of the two derivative couplings of the WZ terms. We discuss briefly our results in section 4, and give a general rule for finding the momentum expansion of any S-matrix element involving the tachyon fields.

2 Three-point function

The three-point amplitude between one RR field and two tachyons in string theory side is given as [20, 23]

$$\mathcal{A}^{T,T,RR} = \left(\frac{i\mu_p}{4}\right) 2\pi \frac{\Gamma[-2u]}{\Gamma[\frac{1}{2}-u]^2} \mathrm{Tr}\,(P_- \not{H}_{(n)} M_p \gamma^a) k_a \ . \tag{10}$$

where $u = -(k+k')^2$ and k, k' are the momenta of the tachyons. In the string theory side we have set $\alpha' = 2$. The trace is zero for $p \neq n$, and for $n = p$ it is

$$\mathrm{Tr}\left(\not{H}_{(n)} M_p \gamma^a\right) = \pm \frac{32}{p!} H_{a_0 \cdots a_{p-1}} \epsilon^{a_0 \cdots a_{p-1} a} \ .$$

We are going to compare string theory S-matrix elements with field theory S-matrix element including their coefficients, however, we are not interested in fixing the overall sign of the amplitudes. Hence, in above and in the rest of equations in this paper, we have payed no attention to the sign of equations. The trace in (10) containing the factor of γ^{11} ensures the following results also hold for $p > 3$ with $H_{(n)} \equiv *H_{(10-n)}$ for $n \geq 5$. The tachyon vertex operator in string theory corresponds to the real components of the complex tachyon of field theory, *i.e.*,

$$T = \frac{1}{\sqrt{2}}(T_1 + iT_2) \tag{11}$$

Now if one replaces k_a in (10) with $-k'_a - p_a$ using the conservation of momentum, one will find that the p_a term vanishes using the totally antisymmetric property of $\epsilon^{a_0 \cdots a_{p-1} a}$. Hence the amplitude (10) is antisymmetric under interchanging $1 \leftrightarrow 2$. This indicates that only the three-point amplitude between one RR, one T_1 and one T_2 is non-zero.

The momentum expansion of (10) is at $u \to 0$. Using the Maple, one can expand the prefactor of (10) around this point, *i.e.*,

$$2\pi \frac{\Gamma[-2u]}{\Gamma[\frac{1}{2}-u]^2} = \frac{-1}{u} + \sum_{n=0}^{\infty} a_n u^u \ . \tag{12}$$

where some of the coefficients a_n are

$$\begin{aligned} a_0 &= 4\ln(2) \\ a_1 &= \frac{\pi^2}{6} - 8\ln(2)^2 \\ a_2 &= -\frac{2}{3}\left(\pi^2 \ln(2) - 3\zeta(3) - 16\ln(2)^3\right) \end{aligned} \tag{13}$$

$$\begin{aligned} a_3 &= \frac{1}{120}\Big(160\pi^2\ln(2)^2+3\pi^4-960\zeta(3)\ln(2)-1280\ln(2)^4\Big) \\ a_5 &= -\frac{1}{90}\Big(160\pi^2\ln(2)^3+9\pi^4\ln(2)-1440\zeta(3)\ln(2)^2+30\zeta(3)\pi^2-768\ln(2)^5-540\zeta(5)\Big) \end{aligned}$$

It is shown in [20] that the massless pole reproduced by the kinetic term of tachyon and the WZ coupling in the first line of (8). There is no higher power of momenta in the massless pole, hence, the kinetic term of tachyon and the WZ coupling have no higher derivative extension. Since the expansion (12) is in terms of the powers of p_a^2, the other terms in (12) correspond to the higher derivative corrections of the WZ action. It is easy to check that the following higher derivative terms reproduce the other terms in (12):

$$2i\alpha'\mu_p\sum_{n=0}^{\infty}a_n\left(\frac{\alpha'}{2}\right)^n C_{p-1}\wedge(D^aD_a)^n(DT\wedge DT^*) \tag{14}$$

The above couplings have on-shell ambiguity, since one can replace T with $\partial^a\partial_a T$ for the on-shell external tachyon. However, we will show in the next section that the above couplings, with exactly the same coefficients a_n, appear in the contact terms as well as in the tachyonic pole of the scattering amplitude of one RR, two tachyons and one gauge field. This indicates that there is no on-shell ambiguity in the above couplings. We shall discus it more in the Discussion section. So, the above couplings are the higher derivative extension of the coupling in the second term in the third line of (8).

The string theory S-matrix element of one RR and two gauge fields is given by [24, 25]

$$\mathcal{A} \sim 2\frac{\Gamma[-2u]}{\Gamma[1-u]^2}K \tag{15}$$

where K is the kinematic factor. Obviously the momentum expansion of this amplitude is around $u\to 0$. Expansion of the prefactor at this point is

$$2\frac{\Gamma[-2u]}{\Gamma[1-u]^2} = -\sum_{n=-1}^{\infty}b_nu^n\ . \tag{16}$$

where some of the coefficients b_n are

$$\begin{aligned} b_{-1} &= 1 \\ b_0 &= 0 \\ b_1 &= \frac{\pi^2}{6} \\ b_2 &= 2\zeta(3) \end{aligned}$$

$$
\begin{aligned}
b_3 &= \frac{19}{360}\pi^4 \\
b_4 &= \frac{1}{3}\left(\zeta(3)\pi^2 + 18\zeta(5)\right) \\
b_5 &= \frac{1}{3024}\left(55\pi^6 + 6048\zeta(3)^2\right)
\end{aligned}
\tag{17}
$$

In this case actually there is no massless pole at $u = 0$ as the kinematic factor provides a compensating factor of u. The amplitude has the following expansion:

$$
\mathcal{A} = i\frac{(4\pi)^2\mu_p}{4(p-3)!} f^{a_0a_1} f'^{a_2a_3} \varepsilon^{a_4\cdots a_p} \epsilon_{a_0\cdots a_p} \left(\sum_{n=-1}^{\infty} b_n u^{n+1}\right) \delta_{p,n+2} \tag{18}
$$

where $f_{ab} = i(k_a\xi_b - k_b\xi_a)$, $f'_{ab} = i(k'_a\xi'_b - k'_b\xi'_a)$ and ε is the polarization of the RR potential. The above terms are reproduced by the following higher derivative couplings of field theory

$$
\frac{\mu_p}{2!}(2\pi\alpha')^2 C_{p-3} \wedge \left(\sum_{n=-1}^{\infty} b_n (\alpha')^{n+1} \partial^{a_1}\cdots\partial^{a_{n+1}} F^{(1)} \wedge \partial_{a_1}\cdots\partial_{a_{n+1}} F^{(1)} - \left(F^{(1)} \to F^{(2)}\right)\right) \tag{19}
$$

We will see in the next section that these couplings, with exactly the same coefficients b_n, appear in the massless pole of the scattering amplitude of one RR, two tachyons and one gauge field. The above couplings are the higher derivative extension of the coupling in the second line of (8).

3 Four-point function

The S-matrix element of one RR field, two tachyons and one gauge field is given as [23]

$$
\begin{aligned}
\mathcal{A}^{ATTC} &= \frac{i\mu_p}{2\sqrt{2\pi}}\Big[\mathrm{Tr}\Big((P_-\not{H}_{(n)}M_p)(k_3.\gamma)(k_2.\gamma)(\xi.\gamma)\Big) I\delta_{p,n+2} + \mathrm{Tr}\Big((P_-\not{H}_{(n)}M_p)\gamma^a\Big) J\delta_{p,n} \\
&\quad \times\Big\{k_{2a}(t+1/4)(2\xi.k_3) + k_{3a}(s+1/4)(2\xi.k_2) - \xi_a(s+1/4)(t+1/4)\Big\}\Big]
\end{aligned}
\tag{20}
$$

where I, J are :

$$
I = 2^{1/2}(2)^{-2(t+s+u)-1}\pi\frac{\Gamma(-u)\Gamma(-s+1/4)\Gamma(-t+1/4)\Gamma(-t-s-u)}{\Gamma(-u-t+1/4)\Gamma(-t-s+1/2)\Gamma(-s-u+1/4)}
$$

$$
J = 2^{1/2}(2)^{-2(t+s+u+1)}\pi\frac{\Gamma(-u+1/2)\Gamma(-s-1/4)\Gamma(-t-1/4)\Gamma(-t-s-u-1/2)}{\Gamma(-u-t+1/4)\Gamma(-t-s+1/2)\Gamma(-s-u+1/4)}
$$

where the Mandelstam variables are

$$
s = -(k_1+k_3)^2, \qquad t = -(k_1+k_2)^2, \qquad u = -(k_2+k_3)^2
$$

k_1 is momentum of the gauge field and k_2, k_3 are the momenta of the tachyons. Note that I, J are symmetric under $s \leftrightarrow t$. The traces in (20) are:

$$\begin{aligned} \mathrm{Tr}\left(H\!\!\!/_{(n)}M_p(k_3.\gamma)(k_2.\gamma)(\xi.\gamma)\right)\delta_{p,n+2} &= \pm\frac{32}{n!}\epsilon^{a_0\cdots a_p}H_{a_0\cdots a_{p-3}}k_{3a_{p-2}}k_{2a_{p-1}}\xi_{a_p}\delta_{p,n+2} \\ \mathrm{Tr}\left(H\!\!\!/_{(n)}M_p\gamma^a\right)\delta_{p,n} &= \pm\frac{32}{n!}\epsilon^{a_0\cdots a_{p-1}a}H_{a_0\cdots a_{p-1}}\delta_{p,n} \end{aligned} \quad (21)$$

Examining the poles of the Gamma functions, one realizes that for the case that $p = n+2$, the amplitude has massless pole and infinite tower of massive poles. Whereas for $p = n$ case, there are tachyon, massless, and infinite tower of massive poles. The tachyon pole in particular indicates that the kinetic term of the tachyon has no higher derivative extension. It has been shown in [23] that the leading order term of the amplitude (20) expanded around the following point :

$$t \to -1/4, \qquad s \to -1/4, \qquad u \to 0 \quad (22)$$

is consistent with the effective actions (1) and (4). We would like to find the field theory couplings which reproduce all terms of the expansion. Let us study each case separately.

3.1 $p = n + 2$ case

For $p = n+2$, the amplitude is antisymmetric under interchanging $2 \leftrightarrow 3$, hence the four-point function between one RR, one gauge field and two T_1 or two T_2 is zero. The electric part of the amplitude for one RR, one gauge field, one T_1 and one T_2 is given by

$$\mathcal{A}^{AT_1T_2C} = \pm\frac{8i\mu_p}{\sqrt{2\pi}(p-2)!}\left[\epsilon^{a_0\cdots a_p}H_{a_0\cdots a_{p-3}}k_{3a_{p-2}}k_{2a_{p-1}}\xi_{a_p}\right]I \quad (23)$$

Note that the amplitude satisfies the Ward identity, *i.e.,* the amplitude vanishes under replacement $\xi^a \to k_1^a$.

Expansion of I around (22) is

$$\begin{aligned} I &= \pi\sqrt{2\pi}\left(-\frac{1}{u}\sum_{n=-1}^{\infty} b_n(s+t+1/2)^{n+1} + \right. \\ &\quad \left. + \sum_{p,n,m=0}^{\infty} c_{p,n,m}u^p\left((s+1/4)(t+1/4)\right)^n(s+t+1/2)^m\right) \end{aligned} \quad (24)$$

where the coefficients b_n are exactly those that appear in (17) and $c_{p,0,0} = a_p$ are those that appear in (13). The constants $c_{p,n,m}$ for some other cases are the following:

$$c_{0,0,2} = \frac{2}{3}\pi^2\ln(2),\ c_{0,1,0} = -14\zeta(3), c_{0,0,3} = 8\zeta(3)\ln(2), \quad (25)$$

$$c_{1,1,0} = 56\zeta(3)\ln(2) - 1/2,\ c_{1,0,2} = \frac{1}{36}(\pi^4 - 48\pi^2\ln(2)^2),\ c_{0,1,1} = -1/2$$

Inserting the first term of (24) into (23), one finds a massless pole which must be reproduced by field theory couplings.

The couplings in (3) and (19), produces the following massless pole for $p = n + 2$:

$$\mathcal{A} \;=\; V_a(C_{p-3}, A^{(1)}, A^{(1)})G_{ab}(A^{(1)})V_b(A^{(1)}, T_1, T_2) \tag{26}$$

where

$$\begin{aligned} G_{ab}(A^{(1)}) &= \frac{i\delta_{ab}}{(2\pi\alpha')^2 T_p\,(u)} \\ V_b(A^{(1)}, T_1, T_2) &= T_p(2\pi\alpha')(k_2 - k_3)_b \\ V_a(C_{p-3}, A^{(1)}, A^{(1)}) &= \mu_p(2\pi\alpha')^2\frac{1}{(p-2)!}\epsilon_{a_0\cdots a_{p-1}a}H^{a_0\cdots a_{p-3}}k_1^{a_{p-2}}\xi^{a_{p-1}}\sum_{n=-1}^{\infty} b_n(\alpha' k_1\cdot k)^{n+1} \end{aligned} \tag{27}$$

where k is the momentum of the off-shell gauge field. Note that the vertex $V_b(A^{(1)}, T_1, T_2)$ has no higher derivative correction as it arises from the kinetic term of the tachyon. The amplitude (26) becomes

$$\mathcal{A} = \mu_p(2\pi\alpha')\frac{2i}{(p-2)!u}\epsilon_{a_0\cdots a_{p-1}a}H^{a_0\cdots a_{p-3}}k_2^{a_{p-2}}k_3^{a_{p-1}}\xi^a\sum_{n=-1}^{\infty} b_n\left(\frac{\alpha'}{2}\right)^{n+1}(s+t+1/2)^{n+1} \tag{28}$$

this is exactly the massless pole of string theory amplitude. Note that there is no left over residual contact term in comparing above amplitude with the massless pole of the string theory amplitude.

The contact terms of string theory amplitude (24) on the other hand are reproduced by the following couplings:

$$\begin{aligned} &2i\alpha'(\pi\alpha')\mu_p\sum_{p,n,m=0}^{\infty} c_{p,n,m}\left(\frac{\alpha'}{2}\right)^p(\alpha')^{2n+m}\,C_{p-3}\wedge\partial^{a_1}\cdots\partial^{a_{2n}}\partial^{b_1}\cdots\partial^{b_m}(F^{(1)}+F^{(2)}) \\ &\wedge(D^aD_a)^pD_{b_1}\cdots D_{b_m}(D_{a_1}\cdots D_{a_n}DT\wedge D_{a_{n+1}}\cdots D_{a_{2n}}DT^*) \end{aligned} \tag{29}$$

For $n = m = 0$ case, the above couplings are the natural extension of the couplings (14) to C_{p-3}. Since there is no on-shell ambiguity for the couplings in (14), one expects there should be no on-shell ambiguity for the above couplings either. The above couplings are the higher derivative extension of the coupling in the fourth line of (8).

3.2 $p = n$ case

Now we consider $n = p$ case. The string theory amplitude in this case is symmetric under interchanging $2 \leftrightarrow 3$. On the other hand, there is no Feynman amplitude in field theory corresponding to four-point function of one RR, one gauge field, one T_1 and one T_2. Hence, for $p = n$ the string theory amplitude (20) is the S-matrix element of one RR, one gauge field and two T_1 or two T_2. Its electric part is,

$$\begin{aligned}\mathcal{A}^{AT_1T_1C} &= \pm\frac{8i\mu_p}{\sqrt{2\pi}p!}\left[\left(\epsilon^{a_0\cdots a_{p-1}a}H_{a_0\cdots a_{p-1}}\right)J\right.\\&\quad\times\left.\left\{k_{2a}(t+1/4)(2\xi.k_3)+k_{3a}(s+1/4)(2\xi.k_2)-\xi_a(s+1/4)(t+1/4)\right\}\right]\end{aligned}\tag{30}$$

Note that the amplitude satisfies the Ward identity, *i.e.,* the amplitude vanishes under replacement $\xi^a \to k_1^a$.

The expansion of $(s+1/4)(t+1/4)J$ around (22) is

$$\begin{aligned}&(s+1/4)(t+1/4)J = \frac{\sqrt{2\pi}}{2}\left(\frac{-1}{(t+s+u+1/2)}+\sum_{n=0}^{\infty}a_n(s+t+u+1/2)^n\right.\\&+\frac{\sum_{n,m=0}^{\infty}d_{n,m}(s+t+1/2)^n((t+1/4)(s+1/4))^{m+1}}{(t+s+u+1/2)}\\&\left.+\sum_{p,n,m=0}^{\infty}e_{p,n,m}(s+t+u+1/2)^p(s+t+1/2)^n((t+1/4)(s+1/4))^{m+1}\right)\end{aligned}\tag{31}$$

where the coefficients a_n in the first line are exactly those appear in (13). Some of the coefficients $d_{n,m}$ and $c_{p,n,m}$ are

$$\begin{aligned}d_{0,0} = -\pi^2/3, &\qquad d_{1,0} = 8\zeta(3)\\d_{2,0} = -7\pi^4/45,\ d_{0,1} = \pi^4/45, &\qquad d_{3,0} = 32\zeta(5),\ d_{1,1} = -32\zeta(5)+8\zeta(3)\pi^2/3\\e_{0,0,0} = \frac{2}{3}\left(2\pi^2\ln(2)-21\zeta(3)\right), &\qquad e_{1,0,0} = \frac{1}{9}\left(4\pi^4-504\zeta(3)\ln(2)+24\pi^2\ln(2)^2\right)\end{aligned}\tag{32}$$

Note that the contact terms in the last line of (31) do not have the structure of the contact terms in the first line of (31). They correspond to different couplings in field theory. The field theory, has the following massless poles for $p = n$:

$$\mathcal{A} = V_a(C_{p-1}, A)G_{ab}(A)V_b(A, T_1, T_1, A^{(1)})\tag{33}$$

where A should be $A^{(1)}$ and $A^{(2)}$. The propagator and vertexes $V_a(C_{p-1}, A)$ are

$$G_{ab}(A) = \frac{i\delta_{ab}}{(2\pi\alpha')^2T_p\,(u+t+s+1/2)}$$

$$
\begin{aligned}
V_a(C_{p-1},A^{(1)}) &= i\mu_p(2\pi\alpha')\frac{1}{p!}\epsilon_{a_0\cdots a_{p-1}a}H^{a_0\cdots a_{p-1}} \\
V_a(C_{p-1},A^{(2)}) &= -i\mu_p(2\pi\alpha')\frac{1}{p!}\epsilon_{a_0\cdots a_{p-1}a}H^{a_0\cdots a_{p-1}}
\end{aligned} \tag{34}
$$

If one uses the kinetic term of the tachyon to find the vertex $V_b(A^{(1)},T_1,T_1,A^{(1)})$ then the amplitude (33) reproduces the massless pole in the first term of (31). To find the higher derivative coupling corresponding to the second term in (31), we consider the following higher derivative terms:

$$
-2\alpha'\mu_p\sum_{n=0}^{\infty}a_n\left(\frac{\alpha'}{2}\right)^n C_{p-1}\wedge(D^aD_a)^n[(F^{(1)}-F^{(2)})|T|^2] \tag{35}
$$

Combining the above with the coupling of one RR, two tachyons and one gauge field of (14), one finds the following coupling:

$$
-2\alpha'\mu_p\sum_{n=0}^{\infty}a_n\left(\frac{\alpha'}{2}\right)^n H_p\wedge(\partial^a\partial_a)^n[(A^{(1)}-A^{(2)})TT^*] \tag{36}
$$

This coupling reproduces exactly the second term in (31). Since the above combination appears naturally in the string theory side, one may expect there there should be no on-shell ambiguity for the couplings in (35) if there is no such ambiguity for the couplings in (14). The couplings in (35) are the higher derivative extension of the coupling in the first term in the third line of (8).

To examine the other terms in the string theory amplitude (30), consider the expansion of $(t+1/4)J$ around (22), *i.e.*,

$$
\begin{aligned}
(t+1/4)J &= \frac{1}{2}\sqrt{2\pi}\Bigg(\frac{-1}{(s+1/4)(t+s+u+1/2)}+\frac{\sum_{n=0}^{\infty}a_n(s+t+u+1/2)^n}{(s+1/4)} \\
&\quad+\frac{\sum_{n,m=0}^{\infty}d_{n,m}(s+t+1/2)^n(t+1/4)^{m+1}(s+1/4)^m}{(t+s+u+1/2)} \\
&\quad+\sum_{p,n,m=0}^{\infty}e_{p,n,m}(s+t+u+1/2)^p(s+t+1/2)^n(t+1/4)^{m+1}(s+1/4)^m\Bigg)
\end{aligned} \tag{37}
$$

Replacing it in the amplitude (30), one finds that the first term of (37) is reproduced by the effective actions (3) and (8). The second term of (37) on the other hand should be reproduced by the following Feynman amplitude in field theory:

$$
\mathcal{A} = V(C_{p-1},T_1,T_2)G(T_2)V(T_2,T_1,A^{(1)}) \tag{38}
$$

where the propagator and the vertex $V(T_2,T_1,A^{(1)})$ which have no higher derivative corrections are given by

$$
\begin{aligned}
V(T_2,T_1,A^{(1)}) &= T_p(2\pi\alpha')(k_3-k)\cdot\xi \\
G(T_2) &= \frac{i}{(2\pi\alpha')T_p(s+1/4)}
\end{aligned} \tag{39}
$$

and the vertex $V(C_{p-1},T_1,T_2)$ should be derived from the higher derivative terms (14), that is

$$
V(C_{p-1},T_1,T_2) = (\alpha')^2\mu_p\sum_{n=0}^{\infty}a_n\left(-\frac{\alpha'}{2}p^ap_a\right)^n\frac{1}{p!}\epsilon^{a_0\cdots a_{p-1}a}H_{a_0\cdots a_{p-1}}k_{2a}
$$

where p^a is the momentum of the RR field. Replacing them in (38), one finds exact agreement with the second term in (37). Note that the couplings (14) appears in (38) as tachyonic pole and as contact term in (36). Moreover, the combination of the tachyonic poles and the contact terms is

$$
\epsilon^{a_0\cdots a_{p-1}a}H_{a_0\cdots a_{p-1}}\sum_{a=0}^{\infty}a_n(s+t+u+1/2)^n\left(\frac{2k_{2a}\xi\cdot k_{3a}}{s+1/4}+\frac{2k_{3a}\xi\cdot k_{2a}}{t+1/4}-\xi_a\right) \tag{40}
$$

which satisfies the Ward identity. This indicates that there is no on-shell ambiguity in the couplings (14). We will discus it more in the Discussion section.

The sum of the massless poles in the second line of (37) and the corresponding term when $k_2\leftrightarrow k_3$, and the massless pole in the second line of (31) is

$$
\begin{aligned}
&4i\mu_p\frac{\epsilon^{a_0\cdots a_{p-1}a}H_{a_0\cdots a_{p-1}}}{p!(s+t+u+1/2)}\left[k_{2a}(t+1/4)(2\xi.k_3)-\frac{1}{2}\xi_a(s+1/4)(t+1/4)+(3\leftrightarrow 2)\right] \\
&\times\sum_{n,m=0}^{\infty}d_{n,m}(s+t+1/2)^n((t+1/4)(s+1/4))^m
\end{aligned} \tag{41}
$$

which satisfies the Ward identity. This should be reproduced in field theory by the amplitude (33) in which the vertex $V_a(C_{p-1},A)$ and the propagator $G_{ab}(A)$ are given in (34) and the vertex $V_b(A,A^{(1)},T_1,T_1)$ should be derived from the the tachyon DBI action and its higher derivative extension in which we are not interested in this paper. In fact it has been checked in [23] that the $d_{0,0}$ term is reproduced by the tachyon DBI action.

Finally, the sum of the contact terms in the last line of (37) and the corresponding term when $k_2\leftrightarrow k_3$, and the last term in (31) is

$$
\begin{aligned}
&4i\mu_p\frac{\epsilon^{a_0\cdots a_{p-1}a}H_{a_0\cdots a_{p-1}}}{p!}\left[k_{2a}(t+1/4)(2\xi.k_3)-\frac{1}{2}\xi_a(s+1/4)(t+1/4)+(3\leftrightarrow 2)\right] \\
&\times\sum_{p,n,m=0}^{\infty}e_{p,n,m}(s+t+u+1/2)^p(s+t+1/2)^n((t+1/4)(s+1/4))^m
\end{aligned} \tag{42}
$$

which satisfies the Ward identity. The field theory couplings corresponding to the above terms are

$$2(\alpha')^2\mu_p\frac{\epsilon^{a_0\cdots a_{p-1}a}H_{a_0\cdots a_{p-1}}}{p!}\sum_{p,n,m=0}^{\infty}e_{p,n,m}(s+t+u+1/2)^p(s+t+1/2)^n((t+1/4)(s+1/4))^m$$
$$\times\Big[\partial_b\partial_c(A^{(1)}-A^{(2)})_aD^bT_1D^cT_1+2D_aD_bT_1D_cT_1\partial^b(A^{(1)}-A^{(2)})^c+T_1\rightarrow T_2\Big]$$

where our notation is such that

$$\begin{aligned}((s+1/4)(t+1/4))^mHATT &\rightarrow (\alpha')^{2m}H\partial_{a_1}\cdots\partial_{a_{2m}}AD^{a_1}\cdots D^{a_m}TD^{a_{m+1}}\cdots D^{a_{2m}}T\\ (s+t+1/2)^nHATT &\rightarrow (\alpha')^nH\partial^{a_1}\cdots\partial^{a_n}AD_{a_1}\cdots D_{a_n}(TT)\\ (s+t+u+1/2)^pHATT &\rightarrow \left(\frac{\alpha'}{2}\right)^pH(D_aD^a)^p(ATT)\end{aligned}\tag{43}$$

Note that the above Lagrangian is invariant under gauge transformation. In terms of field strength, these higher derivative couplings are

$$\begin{aligned}&2(\alpha')^2\mu_p\sum_{p,n,m=0}^{\infty}e_{p,n,m}(s+t+u+1/2)^p(s+t+1/2)^n((t+1/4)(s+1/4))^m\\ &C_{p-1}\wedge\Big[-\partial_b\partial_c(F^{(1)}-F^{(2)})D^bTD^cT^*+2D_bDT\wedge D_cDT^*(F^{(1)}-F^{(2)})^{bc}+\\ &+\partial_b(F^{(1)}-F^{(2)})_c\wedge D^bDTD^cT^*+\partial_b(F^{(1)}-F^{(2)})_c\wedge D^bDT^*D^cT\Big]\end{aligned}\tag{44}$$

Our notation is that the fields without indexes are forms, *e.g.,* $F_c^{(1)}$ is one form and $F^{(1)}$ is two form. The above couplings have on-shell ambiguity which can be fixed by studying the S-matrix element of one RR, two tachyons and two gauge fields in which the above couplings appear in the tachyonic pole and in the contact terms of the amplitude. The above higher derivative couplings have at least four derivatives, so they are not extension of the couplings in (8). This ends our illustration of consistency between the expansion of the S-matrix element of one RR, two tachyons and one gauge field around (22) and the higher derivative couplings of the field theory.

4 Discussion

In this paper, we have shown that the expansion of the S-matrix element of one RR, two tachyons and one gauge field around (22) corresponds to higher derivative extension of the Wess-Zumino terms, *i.e.,* the couplings (29), (35) and (44). Hence, one expects that the expansion (22) to be the momentum expansion, *i.e.,* an expansion that its leading order

terms correspond to the effective action and its non-leading terms correspond to the higher derivative extension of the effective action. In fact, the expansion (22) in terms of momenta of the open string fields is

$$\alpha' k_1 \cdot k_2 \to 0, \, \alpha' k_1 \cdot k_3 \to 0, \, \alpha'(k_2 + k_3)^2 \to 0 \tag{45}$$

The low energy expansion of the amplitude (10) is also around

$$\alpha'(k + k')^2 \to 0 \tag{46}$$

To find a general rule for the momentum expansion of any S-matrix element involving tachyon, let us examine the expansion of some other S-matrix elements. The momentum expansion of the S-matrix element of two gauge fields and two tachyons has been proposed in [26] to be around

$$\alpha' k_1 \cdot k_2 \to 0, \, \alpha' k_1 \cdot k_3 \to 0, \, \alpha' k_2 \cdot k_3 \to 0 \tag{47}$$

where k_1 is momentum of tachyon and k_2, k_3 are the momenta of the gauge fields. The momentum expansion of the S-matrix element of four tachyons has been proposed in [26]. The amplitude has different Chan-Paton factors. The one which has the factor $\mathrm{Tr}\,(l_1 l_2 l_3 l_4)$ should be expanded around

$$\begin{aligned}
&\Big(\alpha'(k_1+k_2)^2, \, \alpha' k_2 \cdot k_3, \, \alpha' k_1 \cdot k_3\Big) \to 0 \\
-&\Big(\alpha'(k_1+k_3)^2, \, \alpha' k_2 \cdot k_3, \, \alpha' k_1 \cdot k_2\Big) \to 0 \\
+&\Big(\alpha'(k_2+k_3)^2, \, \alpha' k_1 \cdot k_2, \, \alpha' k_1 \cdot k_3\Big) \to 0
\end{aligned}$$

The first line produce s-channel, the second line produces u-channel and the last one produces the t-channel. The momentum expansion of the S-matrix element of four tachyons and one gauge field has been proposed in [27]. The amplitude has different Chan-Paton factors and different factors of $k_1 \cdot \zeta_5$, $k_2 \cdot \zeta_5$ and $k_3 \cdot \zeta_5$ where ζ_5 is the polarization of the gauge field. The one which has the factor $\mathrm{Tr}\,(l_1 l_2 l_3 l_4 l_5) k_1 \cdot \zeta_5$ should be expanded around

$$\begin{aligned}
&\Big(\alpha'(k_1+k_2)^2, \, \alpha'(k_3+k_4)^2, \, \alpha' k_2 \cdot k_3, \, \alpha' k_1 \cdot k_5, \, \alpha' k_4 \cdot k_5\Big) \to 0 \\
-&(\alpha' k_1 \cdot k_2, \, \alpha' k_3 \cdot k_4, \, \alpha' k_2 \cdot k_3, \, \alpha' k_1 \cdot k_5, \, \alpha' k_4 \cdot k_5) \to 0 \\
+&\Big(\alpha'(k_2+k_3)^2, \, \alpha' k_3 \cdot k_4, \, \alpha' k_2 \cdot k_3, \, \alpha' k_1 \cdot k_5, \, \alpha' k_4 \cdot k_5\Big) \to 0
\end{aligned}$$

Let us compare the above expansions with the momentum expansion of the S-matrix elements involving only massless fields. The momentum expansion in this case is trivial, *i.e.,*

expansion around $\alpha' k_i \cdot k_j \to 0$ which is equivalent to the expansion around $\alpha'(k_i+k_j)^2 \to 0$. However, for tachyon the expansion around $\alpha' k_i \cdot k_j \to 0$ is not equivalent to the expansion around $\alpha'(k_i+k_j)^2 \to 0$. For example, the expansion of amplitude (10) around $\alpha' k \cdot k' \to 0$ does not produce the massless pole of field theory, hence, that expansion would not be correspond to the effective field theory of brane-anti-brane system. In general, the momentum expansion of a tachyon amplitude should be an expansion around $\alpha' k_i \cdot k_j \to 0$ or $\alpha'(k_i+k_j)^2 \to 0$ or a combination of them for each i, j. The nontrivial question of finding the momentum expansion of a tachyon amplitude in then correspond to fixing this ambiguity.

The above expansions of the tachyon amplitudes are then the expansions in terms of power of momenta of the external states, *i.e.,* α' expansions. Hence, one expects that they should be correspond to the derivative expansions of the field theory. In particular, expansion of the S-matrix element of two tachyons and two gauge fields around (47) should be correspond to the higher derivative coupling of two tachyons and two gauge fields. These higher derivative couplings on the other hand can be used to find the vertex $V_b(A, A^{(1)}, T_1, T_1)$ in the amplitude (33) to produce the massless pole in (41). The details of these calculations will be reported in a forthcoming paper[28], we note here that the constants $d_{n,m}$ that appear in the amplitude (41) have the same structure as the coefficients of expansion of the S-matrix element of two gauge fields and two tachyons [26] expanded around (47), in particular they do not have $\ln(2)'s$. The comparison of the field theory massless pole with the string theory may also give some residual contact terms. These contact terms would have the same structure as those appear in (42), so they would modify the coefficients $e_{p,n,m}$ in (44).

The above rule for expanding the open string tachyon amplitude should be hold even for closed string tachyon amplitude. The expansion of the sphere level S-matrix element of two gravitons and two closed string tachyons in type 0 theory has been proposed in [29] to be around

$$\alpha'(p_3+p_4) \to 0,\ \alpha' p_1 \cdot p_4 \to 0,\ \alpha' p_2 \cdot p_4 \to 0$$

where p_1, p_2 are the graviton momenta and p_3, p_4 are the tachyon momenta. The expansion of the S-matrix element of four closed string tachyons in type 0 theory has been proposed in [29] to be around

$$\frac{1}{3}\left([\alpha'(p_3+p_4),\ \alpha' p_1 \cdot p_4,\ \alpha' p_2 \cdot p_4\right] \to 0$$

$$
\begin{aligned}
&+ [\alpha'(p_1 + p_4),\, \alpha' p_3 \cdot p_4,\, \alpha' p_2 \cdot p_4] \to 0 \\
&+ [\alpha'(p_2 + p_4),\, \alpha' p_3 \cdot p_4,\, \alpha' p_1 \cdot p_4] \to 0)
\end{aligned} \tag{48}
$$

which correspond to s-, t- and u-channels.

The strategy for finding the above expansions in [29] was to find an expansion whose leading order terms are reproduced exactly by the effective action of type 0 theory which includes a covariant tachyon kinetic term. Using this strategy along for finding the expansion of the S-matrix element of two RR and two tachyons, one does not find a unique expansion. In fact in [29] three expansions has been examined which are consistent with the effective field theory. However, neither of them is consistent with the above rule for finding the momentum expansion. A momentum expansion which is also consistent with the strategy in [29] is $(s \to 0,\, t, u \to -1)/2 + (s \to -2,\, t, u \to 0)/2$ which in terms of momentum is

$$
\begin{aligned}
&\frac{1}{2}\,([\alpha'(p_3 + p_4),\, \alpha' p_1 \cdot p_4,\, \alpha' p_2 \cdot p_4] \to 0 \\
&+ [\alpha'(p_1 + p_4),\, \alpha'(p_2 + p_4),\, \alpha' p_3 \cdot p_4,] \to 0)
\end{aligned} \tag{49}
$$

The expansion of the amplitude around this point is

$$
A(C, C, T, T) \;\; \sim \;\; \pi\alpha\left(-\frac{2}{s} - \frac{2}{t} - \frac{2}{u} + 4\ln(2) + \cdots\right) \tag{50}
$$

where α is a factor which has four momenta. The above expansion is consistent with the effective action of type 0 theory and fixes the function $f(T)$ that multiply the kinetic term of the RR fields to be

$$
f(T) \;\; = \;\; 1 + T + \frac{1}{2}T^2
$$

which is the one that has been found in [30].

We have seen that infinite higher derivative couplings in (14) which have been read from the momentum expansion of the S-matrix element of one RR and two tachyons, appear as tachyonic pole of the momentum expansion of the S-matrix element of one RR, two tachyons and one gauge field. This indicates that there should be only one momentum expansion for the string theory S-matrix element of one RR, two tachyons and one gauge field which is consistent with the field theory, *i.e.,* if one expands the amplitude (20) around a point other than (22), one would not find the higher derivative couplings (14) in the tachyonic pole of the amplitude (20). This consistency between different S-matrix element should be holed

for all other S-matrix elements. Therefore, one expects that for any string theory S-matrix element there should be a unique momentum expansion, *e.g.,* the above expansion points should be unique.

The tachyon couplings (14) appear as tachyonic pole and the contact terms of string theory S-matrix element (20) with exactly correct coefficients. The sum of these two terms is gauge invariant. We have interpreted this as an indication that the couplings in (14) have no on-shell ambiguity. To elaborate this point, consider, as an example, the following couplings:

$$2i\alpha'\mu_p\sum_{n=0}^{\infty}a_n\left(\frac{\alpha'}{2}\right)^n(-2\alpha')^2C_{p-1}\wedge(D^aD_a)^n(DD_\alpha D^\alpha T\wedge DD_\beta D^\beta T^*) \tag{51}$$

which is equivalent to (14) for on-shell tachyon. If one considers the above couplings instead of the couplings (14), one would find the contact terms in the second term in (31) after combining them with the couplings in (35). The above couplings produce also the tachyonic pole (38) in which the vertex $V(C_{p-1},T_1,T_2)$ is

$$V(C_{p-1},T_1,T_2) \;=\; (\alpha')^2\mu_p\sum_{n=0}^{\infty}a_n\left(-\frac{\alpha'}{2}p^ap_a\right)^n\frac{1}{p!}\epsilon^{a_0\cdots a_{p-1}a}H_{a_0\cdots a_{p-1}}k_{2a}(2\alpha'k^2)$$

where $k=k_1+k_3$ is the momentum of the off-shell tachyon. If one replaces it into (38), one finds an extra factor of $(2\alpha'k^2)$ in the tachyonic pole. However, one can write it as

$$2\alpha'k^2 \;=\; -4s \;=\; 1-4(s+\frac{1}{4})$$

the first term gives exactly the tachyonic pole which is also produced by (14). When combining it with the above contact terms one finds the gauge invariant combination (40). The second term on the other hand gives an extra contact term. The resulting contact terms are

$$\epsilon^{a_0\cdots a_{p-1}a}H_{a_0\cdots a_{p-1}}\sum_{n=0}^{\infty}a_n(s+t+u+1/2)^n(\xi\cdot k_3\,k_{2a}+\xi\cdot k_2\,k_{3a}) \tag{52}$$

Obviously it does not satisfy the Ward identity, so it can not be reproduced by a gauge invariant coupling in field theory. Therefore, one would find inconsistency between field theory and string theory S-matrix element if one considers the couplings (51) instead of the couplings (14). We expect similar idea should be hole for all other tachyon couplings. In other words, the S-matrix method in principle may have the potential to produce all tachyon couplings without on-shell ambiguity.

Acknowledgment

I would like to thank A. Ghodsi for discussion .

References

[1] M. Gutperle and A. Strominger, JHEP **0204**, 018 (2002) [arXiv:hep-th/0202210]; A. Sen, JHEP **0204**, 048 (2002) [arXiv:hep-th/0203211].

[2] A. Sen, JHEP **0207**, 065 (2002) [arXiv:hep-th/0203265].

[3] A. Sen, Mod. Phys. Lett. A **17**, 1797 (2002) [arXiv:hep-th/0204143].

[4] A. Sen, JHEP **0210**, 003 (2002) [arXiv:hep-th/0207105]; P. Mukhopadhyay and A. Sen, JHEP **0211**, 047 (2002) [arXiv:hep-th/0208142]; A. Strominger, arXiv:hep-th/0209090; F. Larsen, A. Naqvi and S. Terashima, JHEP **0302**, 039 (2003) [arXiv:hep-th/0212248]; M. Gutperle and A. Strominger, Phys. Rev. D **67**, 126002 (2003) [arXiv:hep-th/0301038]; A. Maloney, A. Strominger and X. Yin, JHEP **0310**, 048 (2003) [arXiv:hep-th/0302146]; T. Okuda and S. Sugimoto, Nucl. Phys. B **647**, 101 (2002) [arXiv:hep-th/0208196]; S. J. Rey and S. Sugimoto, Phys. Rev. D **67**, 086008 (2003) [arXiv:hep-th/0301049]; S. J. Rey and S. Sugimoto, Phys. Rev. D **68**, 026003 (2003) [arXiv:hep-th/0303133].

[5] N. D. Lambert, H. Liu and J. M. Maldacena, JHEP **0703**, 014 (2007) [arXiv:hep-th/0303139].

[6] A. Sen, Int. J. Mod. Phys. A **20**, 5513 (2005) [arXiv:hep-th/0410103].

[7] N. Ohta, Phys. Lett. B **558**, 213 (2003) [arXiv:hep-th/0301095].

[8] A. Sen, JHEP **9910**, 008 (1999) [arXiv:hep-th/9909062].

[9] M. R. Garousi, Nucl. Phys. B **584**, 284 (2000) [arXiv:hep-th/0003122].

[10] E. A. Bergshoeff, M. de Roo, T. C. de Wit, E. Eyras and S. Panda, JHEP **0005**, 009 (2000) [arXiv:hep-th/0003221].

[11] J. Kluson, Phys. Rev. D **62**, 126003 (2000) [arXiv:hep-th/0004106].

[12] R. Casero, E. Kiritsis and A. Paredes, arXiv:hep-th/0702155.

[13] O. Bergman, S. Seki and J. Sonnenschein, arXiv:0708.2839 [hep-th].

[14] A. Dhar and P. Nag, arXiv:0708.3233 [hep-th].

[15] T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113**, 843 (2005) [arXiv:hep-th/0412141]; T. Sakai and S. Sugimoto, Prog. Theor. Phys. **114**, 1083 (2006) [arXiv:hep-th/0507073].

[16] A. Sen, JHEP **9808**, 010 (1998) [arXiv:hep-th/9805019]; S. P. de Alwis, Phys. Lett. B **461**, 329 (1999) [arXiv:hep-th/9905080].

[17] M. R. Garousi, JHEP **0501**, 029 (2005) [arXiv:hep-th/0411222].

[18] M. R. Garousi, arXiv:0710.5469 [hep-th].

[19] D. Kutasov, M. Marino and G. W. Moore, arXiv:hep-th/0010108.

M. B. Green, J. A. Harvey and G. W. Moore, Class. Quant. Grav. **14**, 47 (1997) [arXiv:hep-th/9605033].

[20] C. Kennedy and A. Wilkins, Phys. Lett. B **464**, 206 (1999) [arXiv:hep-th/9905195].

[21] P. Kraus and F. Larsen, Phys. Rev. D **63**, 106004 (2001) [arXiv:hep-th/0012198].

[22] T. Takayanagi, S. Terashima and T. Uesugi, JHEP **0103**, 019 (2001) [arXiv:hep-th/0012210].

[23] M. R. Garousi and E. Hatefi, arXiv:0710.5875 [hep-th].

[24] A. Hashimoto and I. R. Klebanov, Phys. Lett. B **381**, 437 (1996) [arXiv:hep-th/9604065].

[25] M. R. Garousi and R. C. Myers, Nucl. Phys. B **542**, 73 (1999) [arXiv:hep-th/9809100].

[26] M. R. Garousi, Nucl. Phys. B **647**, 117 (2002) [arXiv:hep-th/0209068]; M. R. Garousi, JHEP **0312**, 036 (2003) [arXiv:hep-th/0307197];

[27] K. Bitaghsir-Fadafan and M. R. Garousi, Nucl. Phys. B **760**, 197 (2007) [arXiv:hep-th/0607249].

[28] M. R. Garousi and H. Golchin, arXiv:0801.3358 [hep-th].

[29] M. R. Garousi, Nucl. Phys. B **705**, 212 (2005) [arXiv:hep-th/0309028].

[30] I. R. Klebanov and A. A. Tseytlin, Nucl. Phys. B **546**, 155 (1999) [arXiv:hep-th/9811035].